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RATE-OF-CLIMB RECORDER

By Helmut Danielzig

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 849

RATE-OF-CLIMB RECORDER\*

By Helmut Danielzig

SUMMARY

Investigations carried out with this rate-of-climb recorder proved it to be a practical instrument for the accurate recording of static-pressure differences. It can be used for stationary and nonstationary measurements. A heat-insulated glass flask of 4,000-cubic-centimeter content has proved practical as compensating tank.

The chief advantage of the instrument lies in the degree of accuracy obtainable with suitably flexible capsule (dynamic pressure recorder with small test range) and in its sensitivity for recording static-pressure changes.

In contrast to the measurements in which the vertical speeds were determined from the readings of a recording altimeter with a test range of 1,500 meters at the most, the flight measurements can now be executed at any altitude.

For all flight measurements, provision should be made for a time lapse of ~ 25 seconds between the closing of the compensating cock and the actual start of recording in order to bring the flow in the compensating bottle and in the lines to a state of complete rest.

From the appended error calculation, it is seen that no correction of the obtained data, as a result of the pressure change in the compensating vessel relative to sea-level pressure at which it is calibrated, is necessary for heights below 1,000 meters, where at a maximum outside pressure change of as high as 125 millimeters WS, the error is less than 1 percent.

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\* "Steig- und Sinkgeschwindigkeitsschreiber." *Luftwissen*, vol. 4, no. 5, May 1937, pp. 153-157.

## I. INTRODUCTION

The flight performances of an airplane are worked up from climbing and sinking speed measurements. For the determination of these speeds, three principal methods were heretofore available:

- a) The photographic method;
- b) Rate-of-climb indicator reading;
- c) Direct interpretation of the vertical speed from the height record.

The question of employing any one of the three methods must be decided according to the problem under consideration. The first method is rarely used because of its comparatively inferior accuracy and time-consuming evaluation. If it concerns more comprehensive measurements where, for example, several indicating airplane instruments are read or filmed, a rate-of-climb indicator is satisfactory. But, if the time element of some of the recording quantities is involved, the rate-of-climb indicator is ruled out because of its fundamentally limited inertia.

So, of the three methods, the last one remains as the most practical for fundamental investigations, since for research purposes recording instruments are prominently used and the course of the vertical speed can, moreover, be observed for longer periods. Aside from that, the lag of the recording altimeter (so far as optical recording afforded with the Askania quadruple recorder or the DVL dual recorder is employed) is practically nonexistent, because in these instruments the mechanical friction is reduced to an almost irreducible minimum, and the static casing as lag-promoting space, is kept as small as possible. But even this type of speed of climb and descent evaluation is afflicted with various sources of error. One chief source of error lies in the type of test method; the vertical speed of the airplane relative to the ground is measured, whereas what is desired is simply the speed relative to the surrounding air. The ensuing error can be kept small by confining the flight tests to calm weather. Another source of error follows from the inaccuracy of record interpretation. The recording altimeter customarily used in such flight tests, has a range of approxi-

mately 1,500 meters, with a 90-millimeter recording height. The recording periods range from 10 to 15 seconds. Assuming a speed of climb of 3 meters per second, the height of the record in the calibration curve which corresponds to the speed of climb for a 10-second interval, amounts to:

$$\frac{10 \times 3.0 \times 90}{1500} = 1.8 \text{ mm}$$

It is readily seen that this low height of recording can cause very serious errors in the interpretation. If, for reasons of safety, as in spinning investigations, for example, the height is substantially increased, the interpretation is altogether impossible unless the records are photographically enlarged.

Hereinafter follows the description of an instrumental arrangement which enables the reduction of the described interpretation error to a minimum through arbitrary transmission of the height record.

## II. DESCRIPTION AND HOOK-UP OF INSTRUMENT

The instrument (fig. 1) comprises an optical DVL dual recorder which (aside from the height capsule) is fitted with a differential pressure capsule instead of the usual dynamic pressure capsule. The chosen test range of  $\pm 125$  millimeters WS, appears to meet the pressure differences encountered in service. A 4,000 cubic centimeter compensating flask is fitted on the capsule side of the recorder. The static casing of the instruments joins the static lead of the pitot tube in the usual manner. A compensating cock permits connection at any time of the compensating flask with the static circuit to assure pressure balance. At start of recording the compensating cock is closed, trapping a certain amount of air in the pressure capsule and in the compensating flask. As the airplane climbs or descends, the outside pressure changes relatively to the practically constant pressure in the compensating flask.

The pressure is recorded by the pressure-difference recorder and constitutes a criterion for the obtained vertical speeds. For the previously cited capsule with  $\pm 125$  millimeters test range (corresponding reading,  $\pm 45$  millimeters), a 3-meter-per-second vertical speed and a 10-second recording period corresponds to a reading of

$$\frac{10 \times 3 \times 1.36 \times 90}{250} = 14 \text{ mm}$$

With a correspondingly more flexible capsule, still higher readings are obtainable without difficulty, although the capsule described above has proved very reliable for the usual polar measurements, according to the tests.

Between the closing of the compensating cock and the start of the actual measurement, an interval of ~ 30 seconds should elapse to permit the air in the compensating bottle and in the pipes to become perfectly still. During this interval, level flight should be maintained to avoid loss of height recording.

Observance of the pressure is insured through a liquid manometer hooked up, parallel with the pressure capsule. To avoid overstressing in the capsule diaphragm, the liquid manometer is fitted with electric contacts which, on exceeding a certain pressure, are bridged over by the water column made conductive by a slight addition of acid and flash a pilot light. Then the compensating cock must be opened immediately (fig. 2).

The compensating flask is a thermos bottle of 4,000-cubic-centimeter content, packed in thick sponge rubber. The instrument is calibrated after its installation in the airplane. From the calculation which follows, it is seen that the indicating errors caused by calibration at sea-level pressure, are relatively insignificant and negligibly small up to 1,000 meters altitude.

As to the calibration itself, it should be borne in mind that the compensating cock should be opened after each test point in order to prevent a gradual minor temperature change and consequently, erroneous pressure in the compensating flask as a result of volume change in the metering box during calibration and with it an adiabatic change of state. In fact, the calibration with permanently closed compensating cock is accompanied by a slight shift of the neutral line.

For these reasons it is advised not to have the compensating volume less than 4,000 cubic centimeters.

## III. RESULTS OF MEASUREMENTS

The described instrument was mounted in an Albatros L 75 and tested in a series of full-throttle and power-off flights. In conjunction with it, an optical DVL dual recorder for altitude and dynamic pressure was used for comparison. The sinking speeds were evaluated with the new instrument and for the same dynamic pressures also, according to the conventional method through differentiation of the height record. All records were synchronized by means of time marks with a Wetzer contact clock.

The starting of the entire set-up using 6 V was accomplished with a master switch from the observer's seat.

The evaluation of the vertical speed  $w$ , according to the record of the new instrument, follows: If  $\Delta p_a$  denotes the obtained air-pressure difference,  $\gamma_L$  the mean air density, and  $\Delta t$  the recording period in seconds, the relation

$$w = \frac{1}{\gamma_L} \frac{\Delta p_a}{\Delta t} \text{ (m/s)} \quad (1)$$

( $\Delta p_a$  in mm WS)

against the vertical speed from the altitude reading:

$$w = \frac{13.6}{\gamma_L} \frac{\Delta p_a}{\Delta t} \text{ (m/s)} \quad (2)$$

( $\Delta p_a$  in mm Hg)

The validity of both relations rests on the assumption that  $\frac{dp_a}{dt}$  does not change throughout the recording period ( $\sim 12$  seconds). Obviously, this assumption in nowise stipulates a rectilinear course of the record itself, because its characteristic is largely dependent upon the form of the calibration curve (fig. 12). The smaller the compensating flask, the flatter the curve will be, because during the calibration the deflected capsule volume causes an appreciable pressure rise in the compensating flask. To illustrate: For a 1-liter thermos bottle as compensating flask, the pressure rise - with the pressure-difference capsule used in these tests - would already amount to

1.8 mm Hg = 24.4 mm WS for an outside pressure change of 10 mm Hg, according to figure 6. This, however, means that part of the increased evaluation accuracy achieved with the new instrument would be lost again through the then much flatter calibration curve. The check on the constancy of  $\frac{dp_a}{dt}$  follows from the dynamic pressure record; i.e., the dynamic pressure must manifest no change during the recording period. If this is not the case, the differential quotient substitutes for  $\frac{dp_a}{dt}$  at the particular point.

Figure 3 shows various records of speed of climb and descent with the Albatros L 75, along with the vertical speeds obtained by the other two methods for comparison. From the outer boundary curve one can see that the width of the error band representing the rate of climb and descent is much greater when evaluated according to equation (2) than with the new method (equation (1)).

Figure 4 shows the record of the rate-of-climb recorder together with the corresponding record of the altitude recorder at equal dynamic pressures.

The instrument is also practical for recording non-stationary flight movements. But in order to assure satisfactory results, the slow liquid column must be replaced by some indicating member which is not responsive to accelerations, so as to provide adequate protection against overpressures in the recording capsule. In the final version (fig. 2a) this overpressure safeguard is in the form of a statically cased-in pressure-difference capsule which, by means of contacts, closes an electric circuit and flashes a signal light as soon as the test range of the recording-pressure capsule in the optical dual recorder is approached.

Figure 5 shows the static pressure for a disturbance in airplane equilibrium induced by a pull and subsequent release of the elevator (longitudinal oscillation). In contrast to the flat oscillation which corresponds to the same motion of the airplane as record of the altitude recorder, it is possible to discern fine altitude fluctuations on the curve of the new instrument and even to effect a differentiation of the curve.

Some supplementary tests were made in the elevator of the Berlin radio tower, using a pressure-difference capsule

as ordinarily employed for dynamic pressure measurements. The test range amounted to 250 millimeters WS (fig. 1b). Since the elevator speed varied within wide limits, the check on the measuring accuracy was confined to an evaluation of the height. The nominal height of 120.8 meters was ascertained from the instrument reading to within  $\pm 0.8$  meter accuracy, or an error of much less than 1 percent.

#### IV. CALCULATION OF ERROR

##### Notation (fig. 1)

$p_i$ ,	pressure in compensating flask	(mm Hg)
$p_a$ ,	atmospheric pressure	(mm Hg)
$\Delta p_a$ ,	atmospheric pressure change (positive in climb, negative in glide)	(mm Hg)
$\Delta p_i$ ,	pressure change in compensating bottle (positive in climb, negative in glide)	(mm Hg)
$V_i$ ,	volume of compensating bottle	(cm <sup>3</sup> )
$\Delta V_i$ ,	change in volume due to deflection of capsule	(cm <sup>3</sup> )
$a$ ,	factor of capsule	(cm <sup>3</sup> /mm Hg)
$\Delta \Delta p_i$ ,	corrective term	(mm WS)

If the trapped air is left to itself for at least 25 seconds, an isothermal change of state may be assumed, because after that, temperature changes from without or through changes in volume, are practically absent.

With the above notation, we have:

$$p_i V_i = (p_i - \Delta p_i) (V_i + \Delta V_i) \quad (1)$$

hence

$$\Delta p_i = p_i \left( 1 - \frac{V_i}{V_i + \Delta V_i} \right) \quad (2)$$

as change of pressure in the compensating bottle, when the

compensating cock is closed and the change in capsule volume and manometer level is caused by a change in total volume of  $\pm \Delta V_i$ .

Assuming that the pressure change increases in direct proportion to the change in volume and that the pressure on the capsule and manometer is  $(\Delta p_a - \Delta p_i)$ , we can write

$$\Delta V_i = a (\Delta p_a - \Delta p_i)$$

with  $a$  to be defined by test. The simplest way to do this is to fill the capsule with say, alcohol, and load the capsule with the highest occurring pressure difference. The overflowing alcohol which can be measured in a calibrated rising tube connected to the capsule gives, after division by the pressure difference, the capsule coefficient  $a$ . Written in equation (2), we have:

$$\Delta p_i = p_i \left[ 1 - \frac{V_i}{V_i + a (\Delta p_a - \Delta p_i)} \right] \quad (3)$$

which, solved according to  $\Delta p_i$ , gives

$$\Delta p_i = \frac{V_i + a (\Delta p_a + p_i)}{2a} \pm$$

$$\sqrt{\left( \frac{V_i + a (\Delta p_a + p_i)}{2a} \right)^2 - \Delta p_a p_i} \quad (4)$$

This equation (4), the minus sign being applicable herein, can be simplified by applying the approximate equation

$$\sqrt{1 \pm \delta} \approx 1 \pm \frac{\delta}{2}; (\delta \ll 1)$$

Denoting the summand  $\frac{V_i + a (\Delta p_a + p_i)}{2a}$  in equation (4) with  $b$ , gives:

$$\Delta p_i = b - \sqrt{b^2 - \Delta p_a p_i}$$

Then, since  $\frac{\Delta p_a p_i}{b^2} \ll 1$ , we can approximate

$$\Delta p_i = b - b + \frac{\Delta p_a p_i}{2b}$$

$$\Delta p_i = \frac{\Delta p_a p_i}{2b}$$

which, after insertion of  $b$ , leaves:

$$\Delta p_i = \frac{a \Delta p_a p_i}{V_i + a (\Delta p_a + p_i)} \quad (5)$$

For this formula, figure 6 gives for  $a = 0.27$   $\text{cm}^3/\text{mm Hg}$ ,  $p_i = 760 \text{ mm Hg}$ ,  $\Delta p_a = 10 \text{ mm Hg}$ , the course of  $\Delta p_i = f(V_i)$ . For the experimental instrument and its 4,000-cubic-centimeter compensating volume, it means a maximum calibratable error of  $0.49 \text{ mm Hg} = 7 \text{ mm WS}$ . It is not advisable to use less than a 4,000-cubic-centimeter volume, or the calibration curve will become too flat. For the same reason,  $a$  should be kept as small as possible - i.e., the deflected capsule volume also should be kept suitably low.

Forming the total differential of equation (5)

$$d\Delta p_i = dp_i \frac{\partial \Delta p_i}{\partial p_i} + d\Delta p_a \frac{\partial \Delta p_i}{\partial \Delta p_a} \quad (6)$$

the effects of the error can be appraised.

The second summand comprising the change in inside pressure with the outside pressure cancels, since this share of the error is already incorporated in the calibration curve.

Inserting the value for  $\frac{\partial \Delta p_i}{\partial p_i}$  from equation (5), gives: for equation (6),

$$d\Delta p_i = dp_i \frac{\Delta p_a \frac{V_i + a \Delta p_a}{a}}{\left( \frac{V_i + a \Delta p_a}{a} + p_i \right)^2} \quad (7)$$

Since the function  $\Delta p_i = f(p_i)$  represents an unusually flat hyperbole,  $\frac{1}{\Delta p_a} \frac{\partial \Delta p_i}{\partial p_i}$  can be looked upon as being constant within the limits of the flight measurements concerned (0 to 4,000 meters).

For the experimental instrument the value amounts to

$$\frac{1}{\Delta p_a} \frac{\partial \Delta p_i}{\partial p_i} \approx 6.2 \times 10^{-5}$$

This makes the corrective term, which must be subtracted from  $\Delta p_a$  of the calibration curve, after adding finite quantities in equation (7):

$$\Delta \Delta p_i = (p_i - p_a) \Delta p_a 6.2 \times 10^{-5} \text{ (mm WS)} \quad (8)$$

Herein  $p_i$  = ground pressure of calibration (mm Hg),  $p_a$  = pressure at altitude of flight, and  $\Delta p_a$  = outside pressure change (in mm WS) obtained from the calibration curve.

That  $\Delta \Delta p_i$  can be neglected for the conditions encountered in flight operation, is seen from a numerical example.

At 1,000 meters flying altitude,  $(p_i - p_a)$  is ~ 100 mm Hg. The maximum potential  $\Delta p_a$  is  $\pm 125$  mm WS, which leaves:

$$\begin{aligned} \Delta \Delta p_i &= 100 \times 125 \times 6.2 \times 10^{-5} \\ &= \pm 0.77 \text{ mm WS} \end{aligned}$$

to be subtracted from  $\Delta p_a$ .

But if flight measurements are made at greater heights, a correction with respect to equation (8) is absolutely necessary.

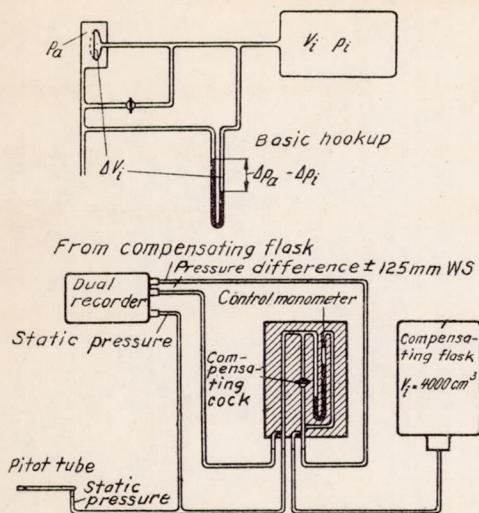


Figure 1.— Basic diagram of experimental instruments.

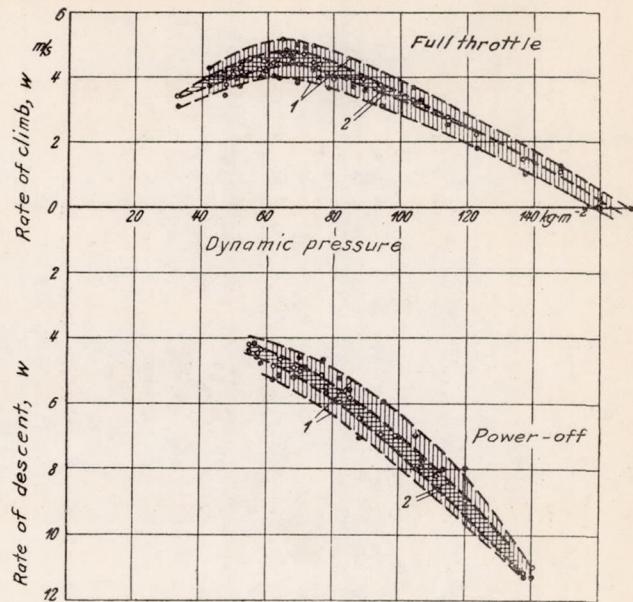


Figure 3.— Determination of rate of climb and descent for the Albatros L75.

- (1) Error band as interpreted from altigraph record,
- (2) Error band as interpreted from rate of climb recorder.

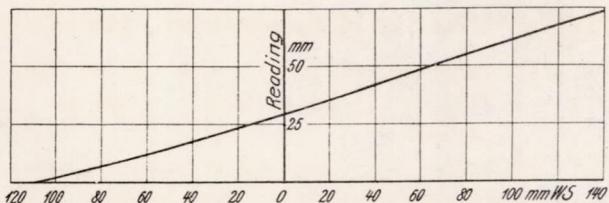


Figure 1a.— Calibration curve from a calibration on the ground.

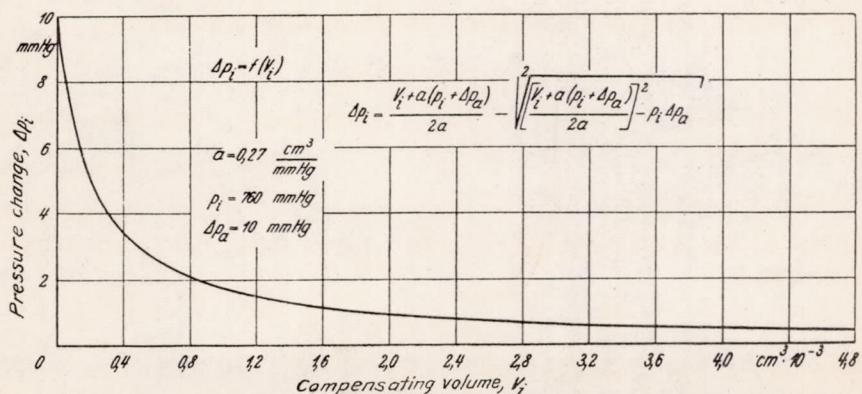


Figure 6.— Pressure change against magnitude of compensating volume.

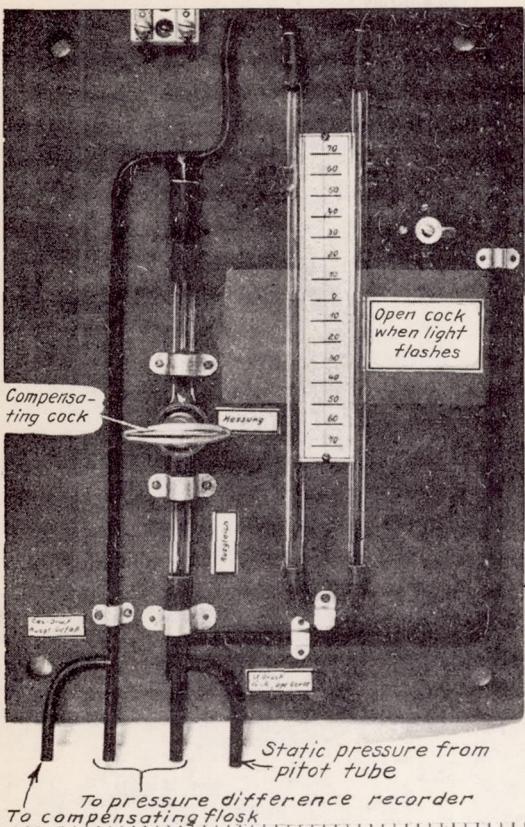


Figure 1b.- Pressure record in the elevator of the Berlin radio tower going up to 120.8 m (396.3 ft.)

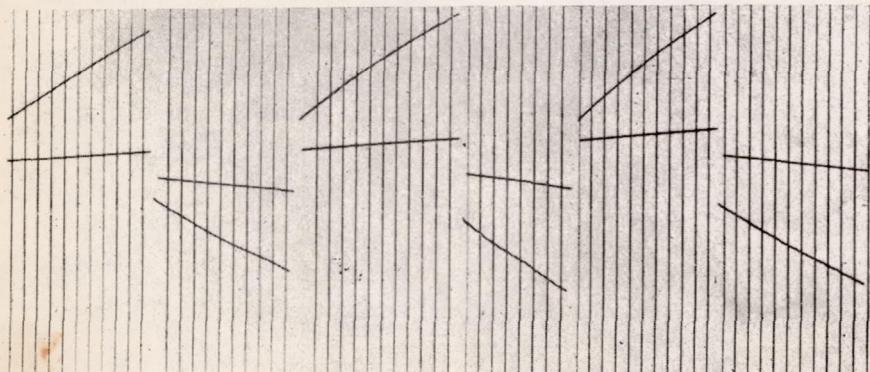


Figure 4.- Record of rate of climb recorder compared to that of recording altimeter for equal dynamic pressure.

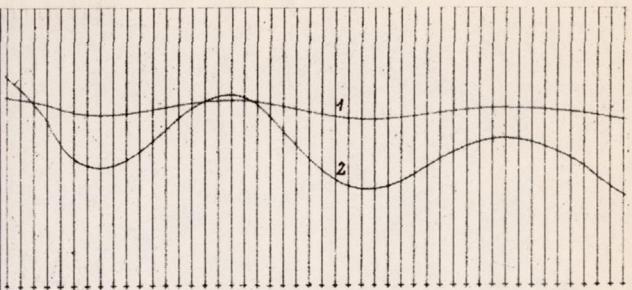


Figure 5.- Record of longitudinal oscillation of airplane due to disturbance caused by elevator.

- (1) Record of recording altimeter.
- (2) Record of rate of climb recorder.

Figure 2.- Control panel for operating rate of climb recorder

Figure 2a.- Final version of instrument with overpressure safeguard (cased-in pressure difference capsule).

